Pressure Drop Through a Tapered Die with Optimized Flow Characteristics

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Synopsis

The flow of power-law liquids through tapered dies has been analyzed in an earlier paper.[1] We now consider a taper which is additionally streamlined so as to make the transition from a broad and sluggish flow to a flow which is narrow and fast and as smooth as possible. This involves (1) the rational selection of an appropriate taper function within the relevant flow geometry and (2) the integration, between limits, of that function.

Introduction

In an earlier paper[1] it was shown how expressions can be derived for the pressure drop during flow in tapered wide-slit and "cylindrical" (strictly speaking, conical) channels.

In polymer melt processing it is common practice to slightly taper the die channel since this has proved to be beneficial with respect to product quality. In particular, the use of cone tapered dies with taper angles not exceeding 10° were found to make it possible to obtain a uniform polymer extrudate with good surface characteristics at head pressures which are up to an order of magnitude higher than the maximum pressure at which an acceptable quality could still be obtained when extruding through a parallel sided (i.e., cylindrical) die.

We now consider the case when the whole of the die entrance region is integrated into the die itself. This is particularly significant in view of the well-known fact that the die entrance is a region where major flow discontinuities due to melt elasticity, partial stagnation flow, and "melt fracture" arise which, unless suppressed during subsequent passage through the die proper, are liable to cause extrudate defects of varying degrees of severity. To minimize or eliminate these defects the processor often has to accept irksome limitations on the output rate.

While die tapers are undoubtedly highly effective in improving the situation, it was considered that an ultimate optimization can only be attained by streamlining the die entrance region as much as possible. This requires the selection of a suitable function which is essentially conicocylindrical and which defines the distance of the wall from the die axis at each point in terms of any given distance along that axis. The axis itself is positioned such as to give the desired entrance and exit radii.

We consider that the die has its optimum shape when the following criteria are met:

1. The tangents to the die wall at the entrance and exit must be parallel to the central axis.

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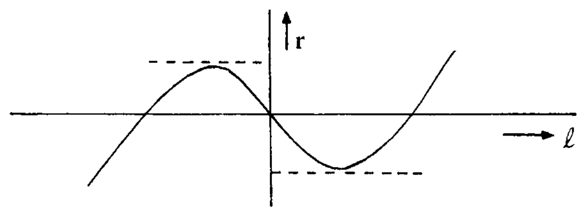

Fig. 1.

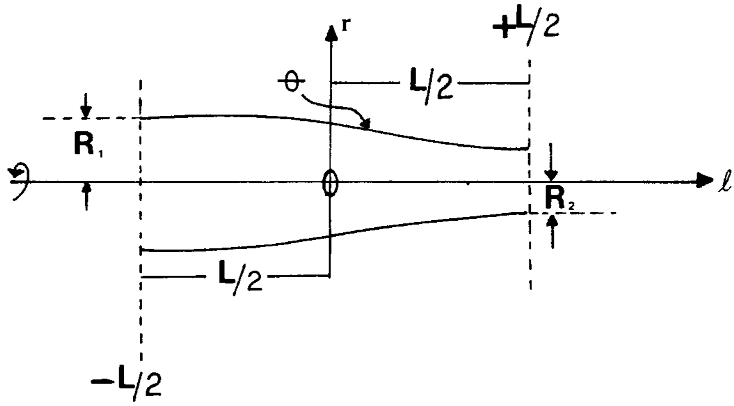

Fig. 2.

2. The longitudinal course of the die wall (i.e., the radius/length function) must be sigmoid and antisymmetrical about a point midway between entrance and exit—that is to say, it has a point of inflexion at what is the origin of the coordinate system in Figure 1.

3. The tangent to the die wall at the midpoint shall be defined by a taper angle θ which is less than 10°. Since the problem requires numerical solution (as will be seen presently), we have arbitrarily selected just three taper angles θ such that $\cot\theta$ has values of 6, 10, and 20 and solved for these angles only, rather than continuously varying values within that range of taper angles.

Solution

We consider that the length/radius function is a quintic of the general form

$$r = al^5 + bl^4 + cl^3 + dl^2 + el + f \tag{1}$$

in which the coefficients b and d are taken to be zero in order to satisfy condition 2 above. We require that this function should have a symmetrical shape as depicted in Figure 1, and this imposes certain conditions on the values of a, c, e, and f. It is seen that this function has a maximum and a minimum. At the origin the tangent to the curve does *not* have a zero slope. We consider this point to be a point of inflexion and stipulate that its slope ($\cot\theta$) should have one of the three numerical values stated above, namely, 6, 10, or 20.

Taking the center portion (i.e., the portion between the maximum and minimum only) and rotating it around an axis parallel with the tangents at the maximum and minimum, we generate a channel with a circular cross section along its entire length, the radius r of which at any point l between entrance and exit

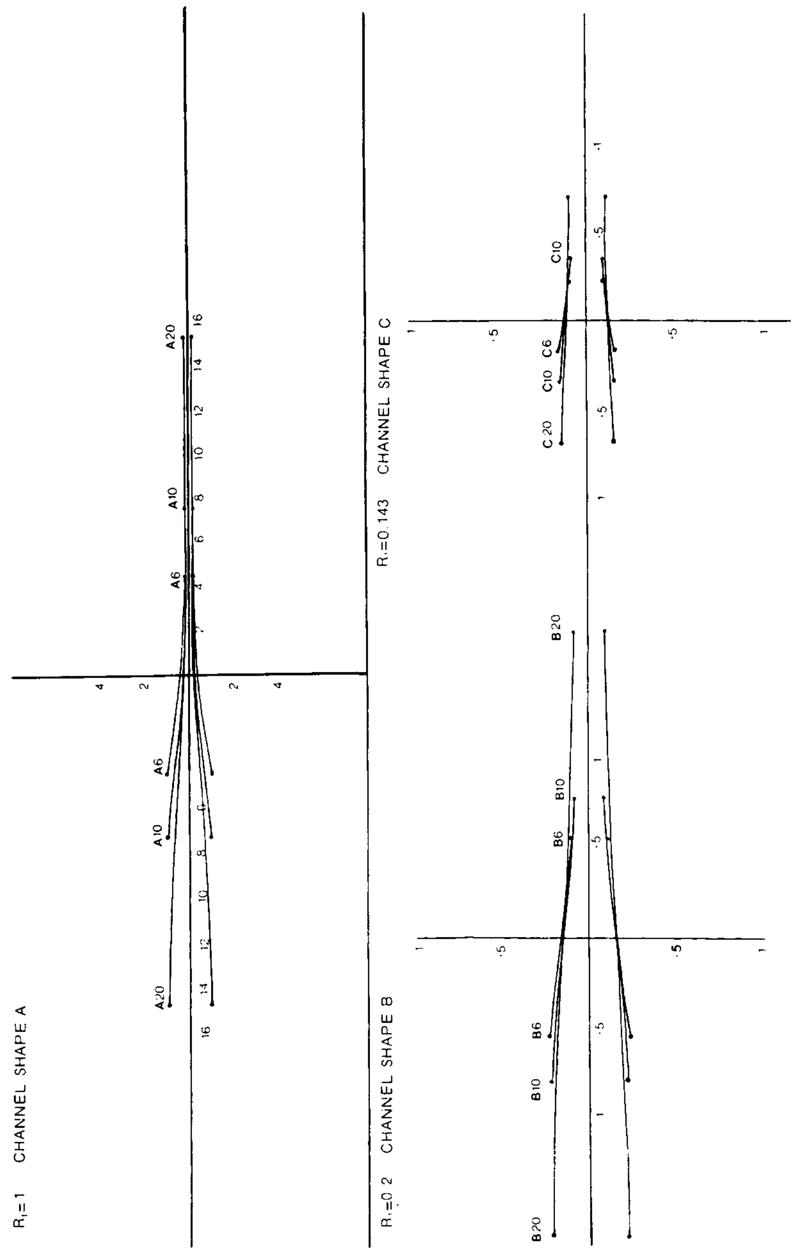

Fig. 3. Channel shapes with $R_2 = 0.1$.

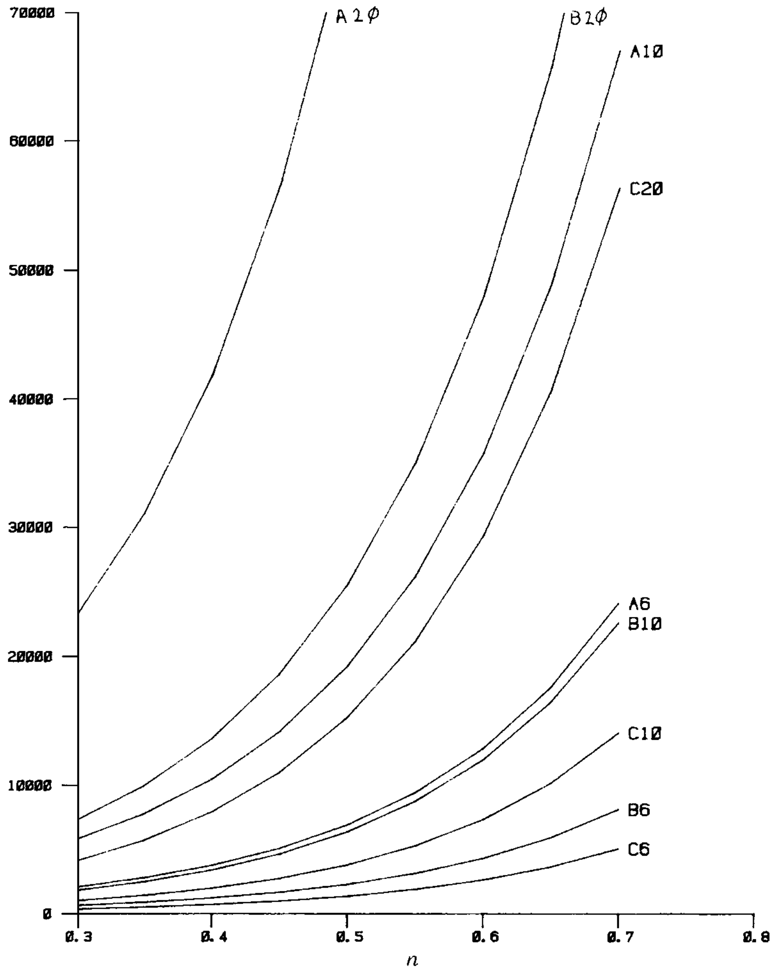

Fig. 4. Graph of $-\Delta P/\eta Q^n$ vs n for channel shapes with $R_2 = 0.1$.

is given by eq. (1). The entrance and exit radii R_1 and R_2, the overall die length L, and the slope at the conventional point of inflexion m are obviously related. In practice, R_1 and R_2 will be fixed by the screw/barrel and extrudate design respectively, and the overall die length L will be determined by the slope m. A die of this type is diagrammatically shown in Figure 2.

As has been seen in the earlier publication,[1] the pressure drop in a truly cylindrical circular channel for a power-law liquid is given by

$$\Delta P = 2\eta L R^{-(3n+1)} \left(\frac{3n+1}{n} \cdot \frac{Q}{\pi}\right)^n \tag{2}$$

where η = coefficient of viscosity, n = the power index (usually between 0.3 and 0.7 in polymer melts), R = the (constant) die radius, and Q = the volume output. If the die radius is not constant, as in cone-tapered dies and in the streamlined dies which are the subject of this paper, then it becomes necessary to consider the pressure gradient dP, given by modifying eq. (2):

$$dP = 2\eta \, dl \, r^{-(3n+1)} \left(\frac{3n+1}{n} \frac{Q}{\pi}\right)^n \tag{3}$$

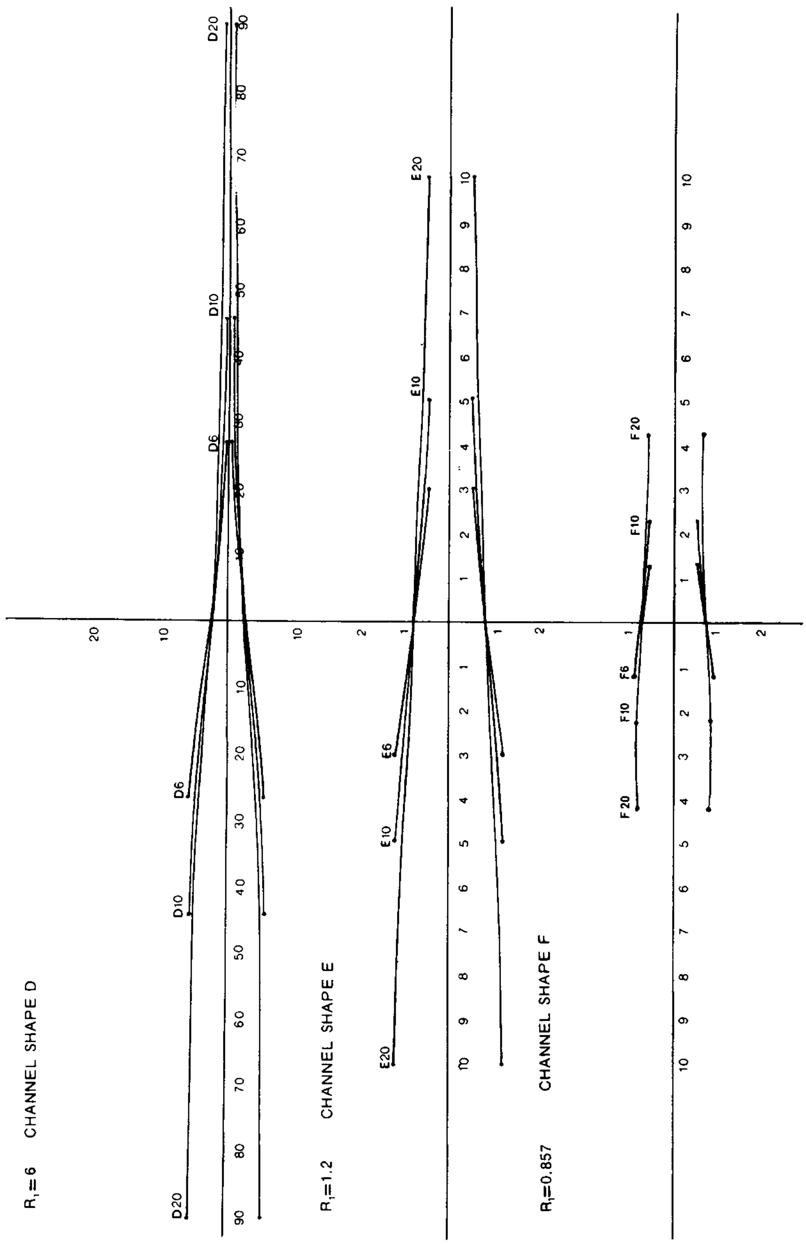

Fig. 5. Channel shapes with $R_2 = 0.6$.

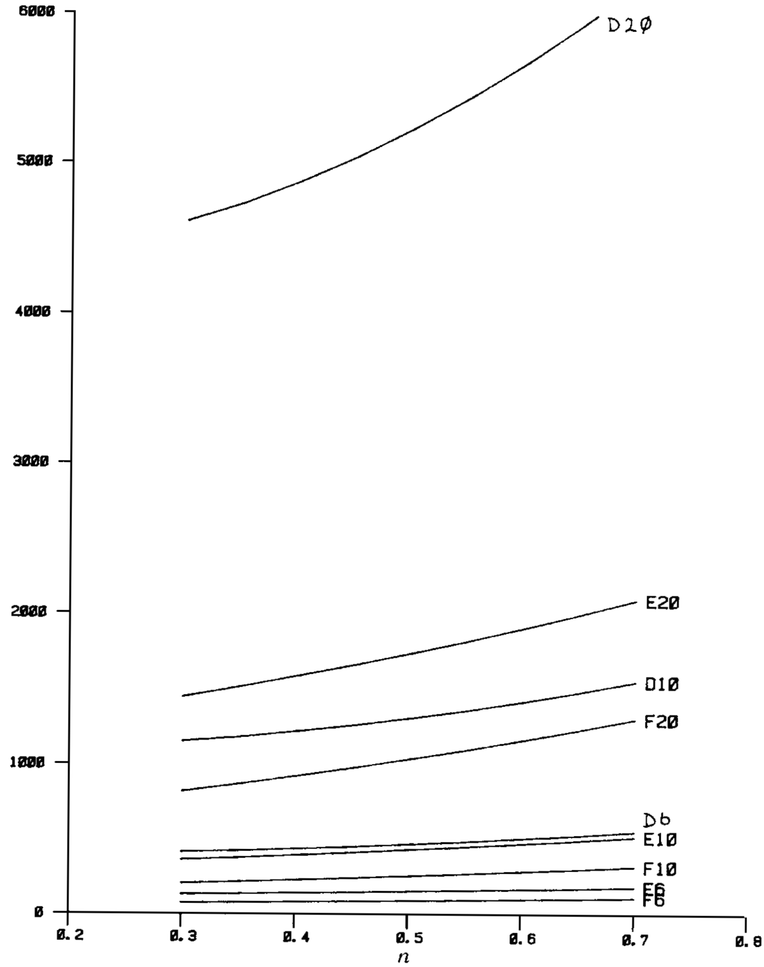

Fig. 6. Graph of $-\Delta P/\eta Q^n$ vs n for channel shapes with $R_2 = 0.6$.

where dl is a simple trigonometric function of r, namely,

$$dl = -dr \cot \theta \tag{4}$$

and r is the radius at any point l along the longitudinal axis between 0 and L.

Substituting into eq. (3) and integrating between the limits of R_1 and R_2 gives the overall pressure drop

$$\Delta P = \frac{2\eta \cot \theta}{3n} \left[\frac{Q(3n+1)}{\pi n}\right]^n R_2^{-3n}\left[1 - \left(\frac{R_2}{R_1}\right)^{3n}\right] \tag{5}$$

This is the solution for cone-tapered dies.

In the streamlined dies which we are now considering, we proceed in the same way in principle, but we encounter difficulties in solving the corresponding integral

$$\int_{-L/2}^{+L/2} r^{-(3n+1)}\, dl \tag{6}$$

that is,

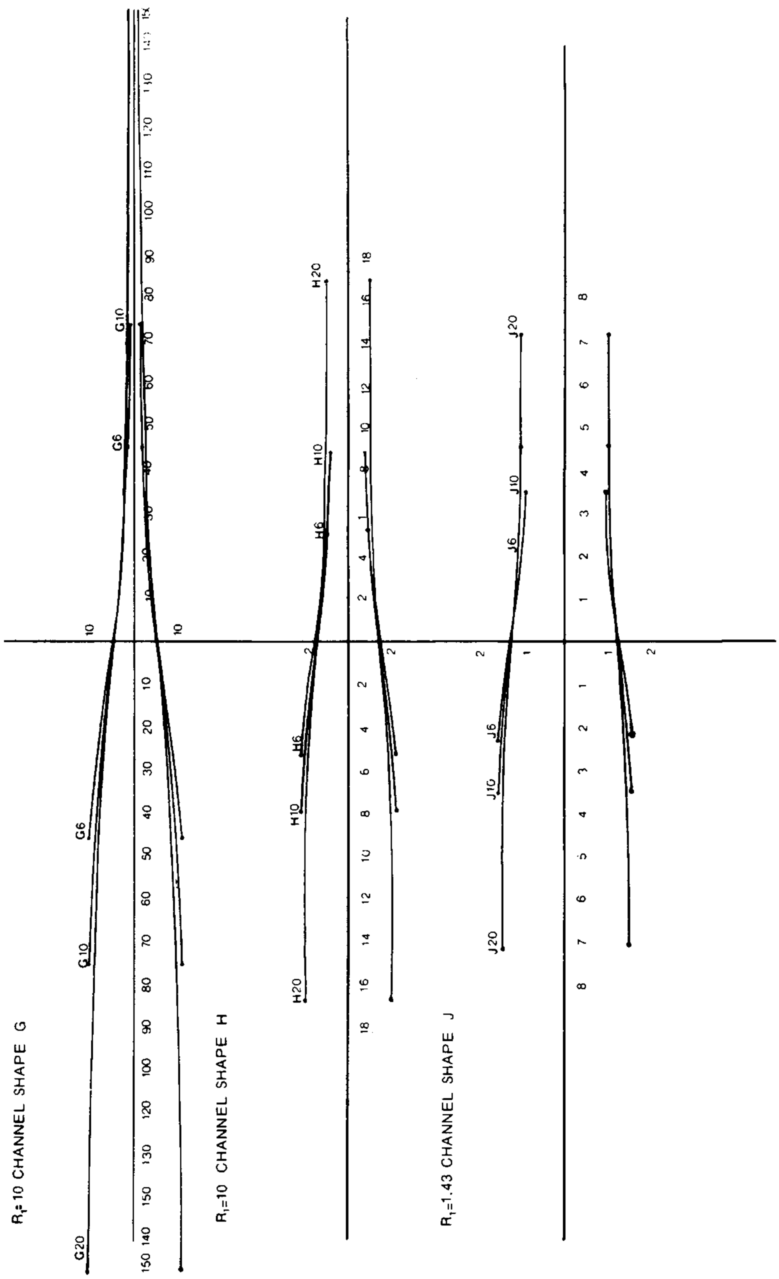

Fig. 7. Channel shapes with $R_2 = 1.0$.

$$\int_{-L/2}^{+L/2} (al^5 + cl^3 + el + f)^{-(3n+1)}\, dl \tag{7}$$

The constants a, c, e, and f must first be found in terms of the given geometric (design) constants. It is also requried that $F(l)$ has no maxima or minima for $-L/2 < l < L/2$:

$$\frac{dr}{dl} = 0 \qquad \text{when} \qquad 5al^4 + 3cl^2 + e = 0$$

Thus,

$$l^2 = \frac{-3c}{10a} \pm \frac{\sqrt{9c^2 + \dfrac{20a}{m}}}{10a}$$

If $a > 0$, one value of l^2 is negative and the other must be $L^2/4$ so that condition (d), defined later, is satisfied. If $a = 0$, then $l^2 = L^2/4$ is a double root, and condition (d) is satisfied.

If $a < 0$, the requirement that there be no solutions for $-L/2 < l < L/2$ means that the smaller of the two solutions for l^2 must be $L^2/4$, and so it is necessary that $L^2/4 < -3c/10a$. Simplifying and letting $y = L/[m(R_1 - R_2)]$ (> 0) gives

$$\frac{1}{4} < \frac{3}{10}\left(\frac{-8y + 10}{24 - 16y}\right)$$

which is

$$0 < \frac{8y - 15}{3 - 2y} \qquad \text{or} \qquad \frac{3}{2} < y < \frac{15}{8}$$

It is thus essential that

$$\frac{3}{2} m(R_1 - R_2) < L < \frac{15}{8} m(R_1 - R_2)$$

We have considered dies for which the value of L comes anywhere within this range. The results are not very sensitive to this small variation in L; and so, in this paper, we take the midpoint value only:

$$L = \frac{27}{16} m(R_1 - R_2)$$

If $F(r) = al^5 + cl^3 + el + f$, the required conditions are: (a) $F(-L/2) = R_1$ is a maximum point; (b) $F(L/2) = R_2$ is a minimum point; (c) at $l = 0$, $dr/dl = -1/m$, where $m = 6, 10$, or 20; (d) $F(l)$ has no maxima or minima for $-L/2 < l < L/2$.

Condition (c) gives $e = -1/m$ and the requirement that $dr/dl = 0$ at $l = \pm L/2$ leads to

$$\frac{5}{16} aL^4 + \frac{3}{4} cL^2 - \frac{1}{m} = 0 \tag{8}$$

Condition (b) also leads to

$$R_2 = \frac{a}{32} L^5 + \frac{c}{8} L^3 - \frac{1}{2m} L + f$$

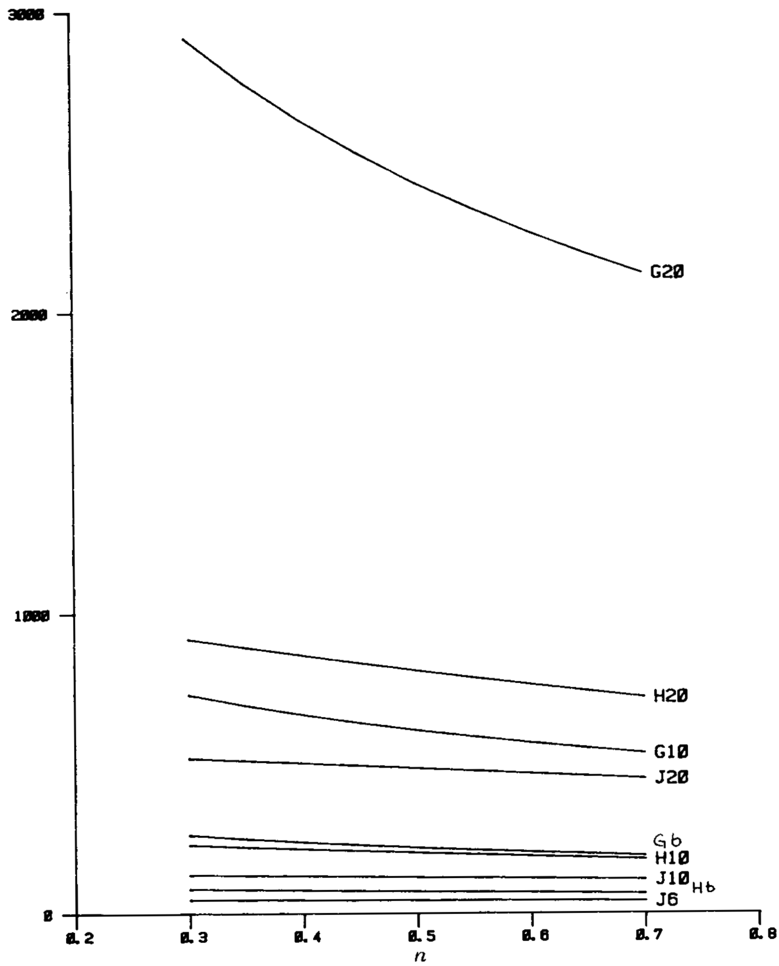

Fig. 8. Graph of $-\Delta P/\eta Q^n$ vs n for channel shapes with $R_2 = 1.0$.

and (a) gives

$$R_1 = -\frac{a}{32}L^5 - \frac{c}{8}L^3 + \frac{1}{2m}L + f$$

Hence, adding

$$R_1 + R_2 = 2f \tag{9}$$

and subtracting,

$$R_2 - R_1 = \frac{a}{16}L^5 + \frac{c}{4}L^3 - \frac{L}{m} \tag{10}$$

Combining eqs. (8) and (10) gives equations for a and c. Thus, we have found a, c, e, and f in terms of R_1, R_2, L, and m:

$$a = \frac{24}{L^5}\left(R_1 - R_2 - \frac{2}{3}\frac{L}{m}\right)$$

$$c = \frac{8}{mL^2} - \frac{10}{L^3}(R_1 - R_2)$$

$$e = -\frac{1}{m}$$

$$f = \frac{1}{2}(R_1 + R_2)$$

The integral (7) cannot be evaluated by analytical methods, and so a numerical approach must be used. Values of 0.1, 0.6, and 1.0 were taken for R_2; and for each of these, R_1 was taken such that

$$R_2 = 0.1R_1; 0.5R_1; 0.7R_1$$

Then, in each of these cases m was taken as 6, 10, and 20. Thus, 27 essentially different channel shapes were considered although our method will apply to any channel in which $6 \leqslant m \leqslant 20$ and $0.1 \leqslant R_2/R_1 \leqslant 0.9$.

The evaluation of (7) has then performed for n = 0.30, 0.50, and 0.70. The method used was the trapezoidal rule together with Romberg's extrapolation method.[2] This technique is available as a subroutine in the IBM Scientific Subroutine Package. The value of

$$2\left(\frac{3n+1}{\pi n}\right)^n m \int_{-L/2}^{L/2} r^{-(3n+1)}\, dl = -\frac{\Delta P}{\eta Q^n}$$

was found, and graphs were produced of $-\Delta P/\eta Q^n$ versus n for each channel shape. The graphs (see Figs. 3–8) are grouped by outlet radius (R_2) so that there are three sets of graphs corresponding to R_2 = 0.1, 0.6, and 1.0. Each set contains a curve for each of nine different channel shapes and the letters A6, B10, etc., correspond to the channel shapes labeled A, B, etc., with m = 6, 10, ... etc. Notice that the $-\Delta P/\eta Q^n$ scales on the graphs are very different in each of the three sets.

References

1. R. S. Lenk, *J. Appl. Polym. Sci.*, **22,** 1775 (1978); **23,** 1587(E) (1979).
2. S. Filippi, *Mathematik-Technik-Wirtschaft,* **11**(2), 49 (1964).

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